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FRACTIONAL WINDING NUMBERS AND THE U(1) PROBLEM

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We simulate the effective lagrangian description of gauge theories with spontaneous mass generation by considering the chiral Gross–Neveu model embedded in a two-dimensional U(1) gauge theory. It is shown that in this hybrid model the non-vanishing expectation value of $\bar{\psi}\psi$ is due to the contribution of instanton configurations with fractional winding.

1. Introduction and outlook

The so-called U(1)-problem [1] has been subject of much discussion. Several points of view have been advocated for its resolution [2, 3]. In particular, Crewther has repeatedly emphasized [4] that configurations with fractional winding should play a crucial role in the dynamical generation of the quark mass *via* a topological symmetry breaking mechanism.

In order to gain some insight into the topological aspects of this problem, we will consider (as usual) a two-dimensional model in which some of the features of the realistic U(1) problem are present. The first candidate one might be tempted to consider would be QED_2 with flavour [5], but this model, contrary to what is expected in QCD_4 [6], does not exhibit dynamical mass generation. We hope, however, to mimic this mass generation by adding to the QED_2 gauge theory a fermion self-interaction of the chiral Gross–Neveu type [7],

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}\left(i\partial\!\!\!/ + \frac{e}{\sqrt{N}}A\!\!\!/\right)\psi + \frac{g^2}{N}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2\right]. \tag{1.1}$$

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Here the fermions are taken to belong to the fundamental representation of U(N). The factor $1/\sqrt{N}$ in the coupling constants e and g has been introduced for later convenience.

The lagrangian (1.1) may be regarded as a substitute for an effective QCD_4 lagrangian [6]. The corresponding theory shares with QCD_4 the property of U(1) invariance, spontaneous mass generation and existence of gauge-field configurations belonging to non-trivial topological classes. It does not, however, share the property of being SU(N) invariant. Thus, unlike in QCD_4, chiral SU(N) invariance is here explicitly broken by the chiral Gross–Neveu interaction.

In the pure chiral Gross–Neveu (GN) model, the spontaneous mass generation is accompanied by the appearance of zero-mode excitations [7]. Since the U(1) vector and axial vector currents are conserved, this is reminiscent of the familiar Nambu–Goldstone mechanism. In four dimensions this would imply a spontaneous breakdown of the chiral U(1) symmetry. However, in two dimensions zero-mass particles do not exist [8]. We shall therefore refer to them as "would be" Goldstone bosons. Their presence can only be reconciled with the Wightman axioms of positivity if they exponentiate [9]. This is what indeed happens in this model. In exponentiated form the would be Goldstone bosons – and hence ψ^{GN} – still carry the U(1) charge and chiral selection rule, implying $\langle\bar{\psi}^{GN}\psi^{GN}\rangle = 0$. Only after extraction of the U(1) part is one left with a fermion field $\hat{\psi}$ which develops a non-vanishing expectation value $\langle\bar{\hat{\psi}}\hat{\psi}\rangle \neq 0$. Since $\hat{\psi}$ is invariant under U(1)×U(1) there is no need for a spontaneous breakdown of the chiral U(1) symmetry.

The above mechanism has been clearly exposed in a paper by Witten [10]. In operator language (f stands for flavour)

$$\psi_f^{GN}(x) = \exp\left[i\sqrt{\frac{\pi}{N}}\left(\gamma^5\varphi(x) + \int_{x^1}^{x} d\xi^1\,\partial_0\varphi\right)\right]\hat{\psi}_f^{GN}(x)\,, \tag{1.2a}$$

where

$$\hat{\psi}_f^{GN}(x) = \left(\frac{\mu}{2\pi}\right)^{1/2} e^{\frac{1}{4}i\pi\gamma^5} \exp\left\{i\sqrt{\tfrac{1}{2}\pi}\sum_{i_D}\lambda_{ff}^{i_D}\left[\gamma^5\phi_{(x)}^{i_D} + \int_{x^1}^{\infty} D\xi^1\,\partial_0\phi^{i_D}\right]\right\}, \tag{1.2b}$$

and φ, ϕ^{i_D} are the canonically quantized "potentials" of the U(1) and diagonal SU(N) currents, respectively [11, 5]

$$\bar{\psi}^{GN}(x)\gamma_\mu\psi^{GN}(x) = -\sqrt{\frac{N}{\pi}}\,\varepsilon_{\mu\nu}\,\partial^\nu\varphi(x)\,, \tag{1.3}$$

$$\bar{\psi}^{GN}(x)\tfrac{1}{2}\lambda^{i_D}\gamma_\mu\psi^{GN}(x) = -\frac{1}{\sqrt{\pi}}\,\varepsilon_{\mu\nu}\,\partial^\nu\phi^{i_D}(x)\,. \tag{1.4}$$

The ϕ^{i_D} satisfy a coupled sine-Gordon-like system of equations, whereas the φ is a free field:

$$\Box\varphi = 0\,. \tag{1.5}$$

Since the $\phi^{(i)}$ are massive fields, the operator $\hat{\psi}_{\rm f}^{\rm GN}$ no longer carries a selection rule. On the other hand, $\psi_{\rm f}^{\rm GN}$ carries the selection rule of a free massless fermion field transforming under U(1)×U(1). When introducing an additional coupling to a gauge field as in (1.1), only this U(1) part will participate in the electromagnetic interaction: A_μ will acquire a mass *via* the usual Schwinger mechanism and φ will spurionize on the gauge-invariant subspace. The only zero-mass excitations remaining are pure gauge and the chiral U(1) invariance is broken spontaneously. Explicitly, one has

$$\psi_{\rm f}(x)_\alpha = \exp\left[-i\sqrt{\frac{\pi}{N}}\,\tilde{\eta}(x)\right]\exp\left[i\sqrt{\frac{\pi}{N}}\,\gamma^5_{\alpha\alpha}\Sigma(x)\right]\hat{\psi}_{\rm f}^{\rm GN}(x)\sigma_\alpha$$

$$A_\mu(x) = -\frac{\sqrt{\pi}}{e}\,\varepsilon_{\mu\nu}\,\partial^\nu(\Sigma(x)+\eta(x))\,, \qquad \Box\eta = 0\,, \tag{1.6}$$

$$\left(\Box+\frac{e^2}{\pi}\right)\Sigma(x) = 0\,, \qquad \partial_\mu\tilde{\eta} = \tilde{\partial}_\mu\eta\,, \qquad \tilde{\partial}_\mu = \varepsilon_{\mu\nu}\,\partial^\nu\,.$$

The $\tilde{\eta}$-dependent exponential is pure gauge. The only other vestige of the original would be Goldstone boson is the spurionic operator [12]

$$\sigma_\alpha = \exp\left[i\sqrt{\frac{\pi}{N}}\,\gamma^5_{\alpha\alpha}(\varphi(x)+\eta(x)) + i\sqrt{\frac{\pi}{N}}\int_{x^1}^{\infty} {\rm d}\xi^1\,\partial_0(\varphi+\eta)\right], \tag{1.7}$$

which carries the original U(1)×U(1) selection rule. On the gauge-invariant subspace this spurion is reduced to a phase $\exp(i\sqrt{\pi}\theta_\alpha/\sqrt{N})$, now labelling the gauge-invariant vacua [13]. Hence, the addition of an electromagnetic interaction results into a "transmutation" of the would-be Goldstone boson of the chiral GN model into a spurion. It follows from this that $\langle\theta|\bar{\psi}\psi|\theta\rangle \neq 0$. Had we not included the GN interaction, the fields carrying the SU(N) degrees of freedom would also have remained massless, and the corresponding selection rules would have prevented $\bar{\psi}\psi$ from developing a vanishing expectation value.

Already, from the operator solution (1.6) we may learn about the gauge classes that will play a role in the corresponding functional formulation: Indeed, on the gauge invariant subspace, $\sigma_2^\dagger\sigma_1$ is equivalent to the operator $T[A]$ inducing non-trivial gauge transformations [12]:

$$T[A^{[1/N]}] = \sigma_2^\dagger\sigma_1\,, \qquad \text{on } \mathcal{H}_{\rm phys}\,,$$

$$A^{[1/N]}(t,\infty) - A^{[1/N]}(t,-\infty) = \frac{2\pi}{N}\,, \qquad \Box A = 0\,.$$

From here we infer that the relevant gauge-field configurations for our model will carry fractional winding.

Spurionic operators of the form (1.7) already appear in the operator solution of the pure Schwinger model with flavour, but due to the SU(N) selection rules mentioned above, only $(\sigma_2^\dagger\sigma_1)^N$ generate the vacua of this model [5]. As a consequence, the pure

Schwinger model does not exhibit fractional winding numbers, except for the meron-like configurations related to charge spurionization [12].

In sects. 2, 3, we shall explicitly exhibit the above outlined mechanism in the Feynman path formulation. A consideration of the problem in the leading order of an $1/N$ expansion is left for sect. 4. We conclude with some remarks.

2. Fractional winding in a path-integral approach

In this section we want to exhibit directly the relevant A_μ configurations in a Feynman path formulation. In order to make the parallelism with respect to the operator discussion as close as possible, we shall use the (euclidean) path-integral approach in the equivalent bosonized form [14]. In order to identify the winding class of the configurations responsible for the dynamical mass generation with a spontaneous breakdown of the U(1) symmetry without Goldstone bosons, we consider the expectation value

$$\langle\theta|J_\pm(x)|\theta\rangle = \langle J_\pm(x)\rangle_{SU(N)} N^{-1} \int d[A_\mu^T]\, d[\varphi] \exp\left[\pm 2i\sqrt{\frac{\pi}{N}}\,\varphi(x)\right] \exp\left[-\int \mathscr{L}_{U(1)}\right], \tag{2.1a}$$

$$\langle J_\pm(x)\rangle_{SU(N)} \sim \int \Pi\, d[\phi^{i_D}]\left\{\frac{1}{N}\sum_f \exp\left[\pm i\sqrt{2\pi}\sum_{i_D} \lambda_{ff}^{i_D}\phi^{i_D}(x)\right]\right\} \exp\left[-\int \mathscr{L}_{SU(N)}\right], \tag{2.1b}$$

where $J_\pm$ is the chirality ± 2 operator

$$J_\pm = \frac{1}{N}\sum_f \bar{\psi}_f \frac{1\pm\gamma^5}{2}\psi_f\,, \tag{2.2}$$

a multiplicative renormalization prescription being understood, and

$$\mathscr{L}_{U(1)} = \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + \tfrac{1}{2}\partial_\mu\varphi\,\partial_\mu\varphi + \frac{ie}{\sqrt{\pi}}(\varepsilon_{\mu\nu}\,\partial_\nu\varphi)A_\mu - \frac{ie}{\sqrt{N}}\frac{\theta}{4\pi}\varepsilon_{\mu\nu}F_{\mu\nu} \tag{2.3a}$$

$$\mathscr{L}_{SU(N)} = \frac{1}{2}\sum_{i_D}\partial_\mu\phi^{i_D}\,\partial_\mu\phi^{i_D} + \frac{g^2}{N}\frac{\mu^2}{\pi^2}\left\{\left(\sum_f \cos\sqrt{2\pi}\sum_{i_D}\lambda_{ff}^{i_D}\phi^{i_D}\right)^2 + \left(\sum_f \sin\sqrt{2\pi}\sum_{i_D}\lambda_{ff}^{i_D}\phi^{i_D}\right)^2\right\}. \tag{2.3b}$$

Hence, in the bosonized language the U(1) part decouples completely from the SU(N) part in the lagrangian. The U(1) part is just the bosonized version of the Schwinger model lagrangian; it involves a zero-mass scalar field φ. $\mathscr{L}_{SU(N)}$ is the SU(N) part of the chiral GN lagrangian describing the interaction of $N-1$ massive scalar fields of the sine–Gordon type. The factorization (2.1) into a Schwinger and

SU(N) chiral GN part is in fact a general property of all correlation functions in this model.

$\langle J_\pm\rangle_{SU(N)}$ can be computed in a $1/N$ expansion (see sect. 4). On the other hand, the U(1) part in (2.1a) may be integrated explicitly. One could first integrate with respect to A_μ; this would lead to an effective lagrangian now involving a massive scalar field φ, thus explicitly exhibiting the loss of the chiral selection rule. However, if one wants to identify the winding configurations, one should first integrate with respect to the fermion degree of freedom φ; this leaves one with, after renormalization,

$$\langle\theta|J_\pm(x)|\theta\rangle = \langle J_\pm\rangle_{SU(N)}\int d[A^T_\mu]\exp\left[-\int d^2z\tfrac{1}{4}F_{\mu\nu}(z)F_{\mu\nu}(z)\right]$$
$$\times\exp\left[i\theta\frac{e}{\sqrt{N}}\int\frac{d^2z}{4\pi}\varepsilon_{\mu\nu}F_{\mu\nu}(z)\right]$$
$$\times e^{-\Gamma[A]}\exp\left[\mp\frac{e}{\sqrt{N}}\int d^2zD(x-z)\varepsilon_{\mu\nu}F_{\mu\nu}(z)\right], \tag{2.4}$$

where $-\Gamma[A]$ is just the logarithm of the fermion determinant in the Schwinger model [15]

$$\Gamma[A]=\frac{e^2}{8\pi}\iint d^2z\, d^2z'\varepsilon_{\mu\lambda}F_{\mu\lambda}(z)D(z-z')\varepsilon_{\nu\rho}F_{\nu\rho}(z'), \tag{2.5}$$

with

$$\Box D(z)=-\delta^2(z), \qquad D(z)=-\frac{1}{4\pi}\ln\mu^2z^2. \tag{2.6}$$

The remaining integration in (2.4) can also be done explicitly; it is saturated by the field configuration [16]

$$A_\mu^{(\pm 1/N)}(z)=\pm\frac{2\pi}{N}\left(\frac{\sqrt{N}}{e}\right)\varepsilon_{\mu\nu}\frac{\partial}{\partial z_\nu}\mathscr{D}(x-z),$$
$$\mathscr{D}(z)=D(z)-\Delta\left(z;\frac{e^2}{\pi}\right), \tag{2.7}$$
$$\left(\Box+\frac{e^2}{\pi}\right)\Delta\left(z;\frac{e^2}{\pi}\right)=-\delta^2(z),$$

which carries a fractional winding number $\nu=\pm 1/N$; it is responsible for the non-vanishing expectation value of $\bar\psi\psi$. This is the mechanism envisaged by Crewther [4] for yielding $\langle\bar\psi\psi\rangle\neq 0$ in QCD$_4$:

$$\Delta Q_5=2\nu N.$$

Fractional winding numbers also occur in the Schwinger model if the fermions are considered to have non-conventional spin [14]; here we see them arise in an entirely conventional canonical theory.

3. Conventional path-integral approach

For the benefit of the reader who would like to see the results of sect. 2 rederived in a conventional path-integral framework, we consider the (Minkowski-space) two point function

$$\langle 0|TJ_-(x)J_+(y)|0\rangle = N^{-1}\int d[A_\mu^T]\,d[\psi]\,d[\bar\psi]J_-(x)J_+(y) \times\exp\left[i\int d^2z\left[-\tfrac{1}{4}F_{\mu\nu}(z)F^{\mu\nu}(z)+\mathscr{L}_{GN}+\frac{e}{\sqrt{N}}j_\mu(z)A^\mu(z)\right]\right], \tag{3.1}$$

where $\mathscr{L}_{GN}$ is the usual chiral GN lagrangian and $N=\langle 0|0\rangle$.

The clustering properties of the two-point function (3.1) will signalize the breakdown of the U(1) symmetry. We can rewrite (3.1) as follows:

$$\langle 0|TJ_-(x)J_+(y)|0\rangle = N^{-1}\int d[A_\mu^T]\exp\left[-i\int d^2z\tfrac{1}{4}F_{\mu\nu}(z)F^{\mu\nu}(z)\right]Z[A], \tag{3.2}$$

where

$$Z[A]\equiv\left\langle TJ_-(x)J_+(y)\exp\left[\frac{ie}{\sqrt{N}}\int d^2zj_\mu(z)A^\mu(z)\right]\right\rangle_{GN} \tag{3.3}$$

is the external field correlation function of the chiral GN model. Regarding $Z[A]$ as a generating functional, we obtain from it all correlation functions with an arbitrary number of current insertions.

The A_μ dependence of $Z[A]$ may be computed by observing that the current is, of course, conserved and the axial current in an external potential has – as a result of the asymptotic freedom of the model – the usual anomaly:

$$\partial_\mu j^\mu(x)=0\,,\qquad \partial_\mu j_5^\mu(x)=-\frac{N}{2\pi}\frac{e}{\sqrt{N}}\varepsilon_{\mu\nu}F^{\mu\nu}(x)\,. \tag{3.4}$$

Functional differentiation of $Z[A]$ with respect to A_μ gives

$$\frac{\delta Z[A]}{\delta A_\mu(z)}=\frac{ie}{\sqrt{N}}\langle TJ_-(x)J_+(y)j^\mu(z)\rangle_{GN,A}\,. \tag{3.5}$$

Using (3.4) and the equal-time commutation relations

$$[j^0(x),J_\pm(0)]_{ET}=0\,,\qquad [j_5^0(x),J_\pm(0)]_{ET}=\pm 2J_\pm(x)\,\delta(x^1)\,, \tag{3.6}$$

we obtain from (3.5)

$$\partial_\mu\frac{\delta}{\delta A_\mu(z)}Z[A]=0\,,\qquad \tilde\partial_\mu\frac{\delta}{\delta A_\mu(z)}Z[A]=2i\frac{e}{\sqrt{N}}Z[A][\delta^2(z-x)-\delta^2(z-y)]-i\frac{e^2}{\pi}Z[A]\tilde\partial_\mu A^\mu(z)\,. \tag{3.7}$$

These equations have the solution

$$Z[A]=Z[0]\exp\left[-i\frac{e^2}{2\pi}\iint d^2z\, d^2z'\,\tilde{\partial}_\mu A^\mu(z)D_c(z-z')\,\tilde{\partial}_\nu A^\nu(z')\right]$$
$$\times\exp\left[2i\frac{e}{\sqrt{N}}\int d^2z(D_c(x-z)-D_c(y-z))\,\tilde{\partial}_\mu A^\mu(z)\right], \tag{3.8}$$
$$\Box D_c(z)=-\delta^2(z),$$

where evidently

$$Z[0]=\langle 0|\mathrm{T}J_-(x)J_+(y)|0\rangle_{\mathrm{GN}}. \tag{3.9}$$

$Z[A]$ still contains zero-mass excitations which are known to factorize exactly [see also eq. (1.2)]; after renormalization

$$\langle 0|\mathrm{T}J_-(x)J_+(y)|0\rangle_{\mathrm{GN}}=\left[\frac{1}{-\mu^2(x-y)^2+i0}\right]^{1/N}\langle 0|\mathrm{T}J_-(x)J_+(y)|0\rangle_{\mathrm{SU}(N)}\,. \tag{3.10}$$

Combining (3.10), (3.9) and (3.8) in (3.2) one sees that, as in the case of ref. [16], the euclidean A_μ integration is saturated by the classical configuration

$$A_\mu(z)=\frac{2\pi}{N}\left(\frac{\sqrt{N}}{e}\right)\varepsilon_{\mu\nu}\frac{\partial}{\partial z_\nu}[\mathscr{D}(x-z)-\mathscr{D}(y-z)]\,, \tag{3.11}$$

corresponding to a pair of an (fractional winding) instanton and anti-instanton. Thus we obtain

$$\langle 0|J_-(x)J_+(y)|0\rangle=\langle 0|J_-(x)J_+(y)|0\rangle_{\mathrm{SU}(N)}\exp\left[\frac{4\pi}{N}\Delta\left(x-y;\frac{e^2}{\pi}\right)\right]. \tag{3.12}$$

Therefore, the whole effect of the A_μ integration was to turn the massless U(1) factor in (3.10) into a massive one. The SU(N) factor can be computed in a $1/N$ expansion to be (see sect. 4)

$$\langle 0|J_-(x)J_+(y)|0\rangle_{\mathrm{SU}(N)}=\tfrac{1}{4}m^2+\frac{1}{N}f(x-y)+\cdots\,, \tag{3.13}$$

where $f(z)$ vanishes for $z^2\to\infty$. Hence, one obtains *via* clustering the tunneling amplitude

$$\left\langle\frac{1}{N}\middle|J_+(x)|0\right\rangle=\tfrac{1}{2}m\,, \tag{3.14}$$

in accordance with the results of sect. 2.

4. 1/*N* expansion

The result (3.8), where the vector potential appears in exponential form, provides a direct insight into the topological structure of this model. If one were to expand in powers of $e/\sqrt{N}$, this structure would become hidden, and the only remnant of the θ-dependence would be a hidden long-range force – in the sense of ref. [17] – in the correlation function $\langle A_\mu J_\pm \rangle$. The $1/N$ expansion would thus reflect the non-trivial topological aspects of this model only in this indirect sense.

The investigation of a large class of two-dimensional models [7, 18, 19, 20] lends support to the validity of a $1/N$ expansion. Nevertheless, the usual chiral GN model only admits such an expansion for the SU(N) symmetric part of the fermionic correlation functions because of the infraparticle structure associated with the U(1) would be Goldstone boson. [10, 19]. This is expected not to be true for the chiral GN model embedded in a U(1) gauge theory, where the gauge field serves to screen the U(1) × U(1) charge of the GN fermions, thus dynamically reducing them to SU(N) multiplets. Hence the only massless excitations remaining in the model will be pure gauge. It is thus instructive to see how this manifests itself in leading order of $1/N$.

For simplicity we shall restrict ourselves again to the gauge-invariant two-point correlation function (3.1) in which no trace of zero-mass excitations should remain.

Introducing as usual the auxiliary fields σ and π, where π corresponds to $\bar{\psi} i\gamma^5 \psi$ and σ corresponds to $\bar{\psi}\psi - Nm$, with m the spontaneously generated mass of the *ordinary* GN model determined in leading order by the condition

$$m = g^2 \operatorname{tr} \int \frac{d^2 p}{(2\pi)^2} \frac{i}{\not{p} - m}, \tag{4.1}$$

one obtains, after integrating out the fermions, the effective action in leading order of $1/N$,

$$S_{\text{eff}} = \frac{1}{2} \int\int [\sigma \Gamma_\sigma \sigma + \pi \Gamma_\pi \pi + 2\sigma \Gamma_\mu A^\mu + 2\pi \Gamma^5_\mu A^\mu + A^\mu \Gamma_{\mu\nu} A^\nu]. \tag{4.2}$$

Here the various vertices have the graphical representations shown in fig. 1. A

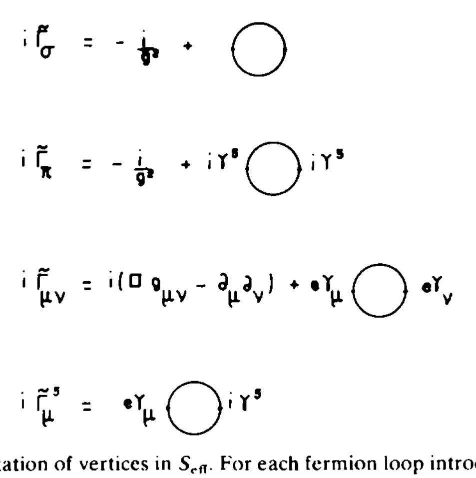

Fig. 1. Graphical representation of vertices in S_{eff}. For each fermion loop introduce an extra minus sign.

straightforward calculation yields for the corresponding Fourier transforms

$$\tilde{\Gamma}_\sigma(p) = -\frac{1}{2\pi}\frac{\varphi}{\tanh\frac{1}{2}\varphi},$$

$$\tilde{\Gamma}_\pi(p) = -\frac{1}{2\pi}\frac{\varphi}{\operatorname{ctnh}\frac{1}{2}\varphi},$$

$$\tilde{\Gamma}_{\mu\nu}(p) = \left[\left(-p^2 + \frac{e^2}{\pi}\right) - 4e^2m^2\frac{\tilde{\Gamma}_\pi(p)}{p^2}\right]\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right), \tag{4.3}$$

$$\tilde{\Gamma}^5_\mu(p) = -2iem\frac{\tilde{p}_\mu}{p^2}\tilde{\Gamma}_\pi(p) \qquad \tilde{\Gamma}_\mu(p) = 0,$$

where φ is the usual rapidity variable,

$$p^2 = -4m^2\sinh^2\tfrac{1}{2}\varphi.$$

$S_{\rm eff}$ is diagonalized by defining a new field

$$\pi' = \pi - 2em\frac{\tilde{\partial}_\mu}{\Box}A^\mu \tag{4.4}$$

and we are left with

$$S_{\rm eff} = \frac{1}{2}\iint\left[\sigma\Gamma_\sigma\sigma + \pi'\Gamma_\pi\pi' + A^\mu\left(\Box + \frac{e^2}{\pi}\right)\left(g_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right)A^\nu\right]. \tag{4.5}$$

Keeping in mind (4.4) we obtain from here the following Feynman rules. *Propagators*:

$$\tilde{\Delta}_{\pi'}(p) = -2\pi i\frac{\operatorname{ctnh}\frac{1}{2}\varphi}{\varphi},$$

$$\tilde{\Delta}_\sigma(p) = -2\pi i\frac{\tanh\frac{1}{2}\varphi}{\varphi},$$

$$\tilde{D}_{\mu\nu}(p) = \frac{i}{-p^2 + e^2/\pi}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right).$$

with the corresponding vertices shown in fig. 2.

Observing that eq. (4.1) gives the vacuum expectation value of J_+, with g^2 playing the role of the multiplicative renormalization constant, one has

$$\langle J_+\rangle = \tfrac{1}{2}m.$$

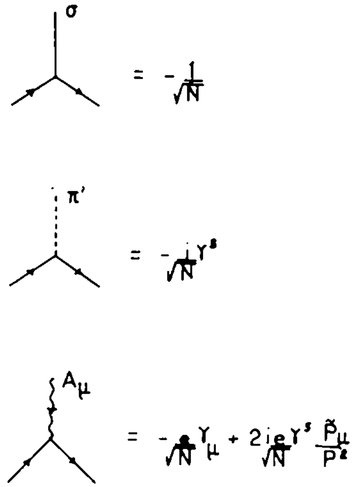

Fig. 2. Feynman rules for vertices.

Using the above Feynman rules one further obtains

$$\int d^2x\, e^{ip\cdot x}\langle TJ_-(x)J_+(0)\rangle = \pi^2 m^2\delta^2(p) - \tfrac{1}{2}i\pi\frac{\tanh\frac{1}{2}\varphi}{\varphi} - \tfrac{1}{2}i\pi\frac{\mathrm{ctnh}\frac{1}{2}\varphi}{\varphi} - \frac{im^2}{p^2}\,\frac{e^2}{-p^2+e^2/\pi}\,,$$

which approaches a finite limit as $p^2 \to 0$, thus showing that no trace of the would be Goldstone mode is left. This is in accordance with the results of sect. 3.

5. Conclusion

The main point of this paper was to gain some insight into the topological nature of the configurations responsible for the simultaneous breakdown of the chiral U(1) symmetry and the dynamical fermion mass generation. In the particular model chosen to study this problem we confirmed Crewther's ideas on the relevance of fractional winding. Of course, in QCD_4 fractional winding configurations are expected [21] to have infinite action; however, the role of finiteness of action is not clearly understood beyond the semiclassical approximation [4, 6].

We have organized the paper by approaching the problem from four different points of view. From the operator point of view it was already clear that the particle structure of this model is the same one as that of the chiral Gross–Neveu model plus an uncoupled plasmon, which plays the role of the η in the U(1) problem. Therefore, in this model charge-screened quarks are liberated, closely paralleling what happens in the massless Schwinger model [5]. Hence it is the zero bare mass of the fermion, and not the dynamically generated one, which plays the decisive role in the "screening *versus* confinement" [22] aspects of the model. The coupling to the gauge field played a stabilizing role in the $1/N$ expansion, insuring that the naive zero-order spontaneous breakdown of the chiral invariance is maintained in the exact solution, contrary to what occurs in the pure chiral Gross–Neveu model.

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